ISSN 1029-9599, Physical Mesomechanics, 2018, Vol. 21, No. 2, pp. 117–123. © Pleiades Publishing, Ltd., 2018.
Original Text © F.S. Alzahrani, I.A. Abbas, 2018, published in Fizicheskaya Mezomekhanika, 2018, Vol. 21, No. 1, pp. 23–29.

Fractional Order Theory in a Semiconductor Medium Photogenerated by a Focused Laser Beam

F. S. Alzahrani[1]* and I. A. Abbas[1,2]

[1] *Nonlinear Analysis and Applied Mathematics Research Group, Department of Mathematics, King Abdulaziz University, Jeddah, 21589 Saudi Arabia*
[2] *Department of Mathematics, Faculty of Science, Sohag University, Sohag, 82524 Egypt*
* *e-mail: faris.kau@hotmail.com*

Received March 27, 2017

Abstract—In this paper, the fractional order theory has been applied for thermal, elastic and plasma waves to determine the carrier density, displacement, temperature and stress in a semiconductor medium. The thermal, elastic and plasma waves in a semi-infinite medium photogenerated by a focused laser beam were analyzed. The Laplace transformation is used to express the governing equation and solved analytically by applying eigenvalue approach methodology in that domain. A semiconducting material like as silicon was considered. According to the numerical results and graphics, the fractional order parameter and thermal relaxation time may play an important role in the behavior of all physical quantities.

DOI: 10.1134/S1029959918020042

Keywords: fractional order theory, photothermoelastic waves, eigenvalue approach, laser beam, Laplace transform

1. INTRODUCTION

When a semiconductor with band gap energy E_g is illuminated by a laser beam with an energy E higher than E_g, an excitation process of electrons will take place. The excited electrons will transfer to an energy level from the valence band with energy of $E - E_g$ above the conduction band edge. Then these photoexcited free carriers will relaxe to one of the unfilled levels nearby the conduction band bottom (transition of nonradiative). After relaxation there are electron and hole plasma which is followed by the formation of hole–electron pairs through the recombination process. In semiconductors there is a periodic elastic deformation produced by the photoexcited carriers known as electronic deformation. The electronic deformation may cause local tensions in the sample which can introduce plasma waves which are similar to the heat wave generated by local periodic elastic deformation. Considerable attention has been currently given to the surface waves of bounded plasma systems. The existence of plasma boundaries makes it possible for various surface wave modes to arise, which may in some cases have frequency spectra drastically different from those of volume-wave modes.

Understanding of transport phenomena in the solid through the development of spatially in situ resolved probes has great of attention. In the present work we try to measuring transport processes based on the principle of optical beam deflection using a photothermal approach that can be considered as an expansion of the photothermal deflection technique. Such a technique is contactless and directly yields the parameters of electronic and thermal transport at the semiconductor surface or at the interface and within the bulk of the semiconductor. Pure silicon is an intrinsic semiconductor and is used widely in semiconductor industry, for example, the monocrystalline Si is used to produce silicon wafers. In general, the conduction in the semiconductor (pure Si) is not the same as in metals. Both the electrons and holes are responsible about the conduction in semiconductors as well as the electrons that may be released from atoms by heat. Therefore electric resistance of semiconductors decreases with temperature increasing.

Todorovic et al. [1–3] discussed theoretical and experimental results on microelectromechanical structures in plasma, thermal and elastic waves. These results give valuable information about carrier recombination and transport mechanisms in semiconductors. Also, the study includes the variations in propagating both plasma and thermal waves due to the linear coupling between heat and mass transport (i.e., thermos diffusion). The effects of thermoelastic and electronic deformations in semiconductors without considering the coupled system of the equations of thermal, elastic and plasma waves have been studied [4–6]. In addition, local thermoelastic deformations at the sample surface due to the excitation by a probe beam have been analyzed by Rosencwaig et al. [7], then Opsal and Rosencwaig [8] study the depth profiling of thermal and plasma waves in silicon. On the other hand, Song et al. [9, 10] study in detail the generalized thermoelastic vibrations due to optically excited semiconducting microcantilevers. They concluded that the plane wave reflection in a semiconducting material is under the theory of generalized thermoelasticity and photothermal theory [11, 12].

Many existing models of physical processes have been modified successfully by using the fractional calculus. Fractional order of weak, normal and strong heat conductivity under generalized thermoelastic theory were established by Youssef [13, 14] who developed the corresponding variational theorem also. The theory was then used to solve the 2D problem of thermal shock using the Laplace and Fourier transforms [15]. Based on a Taylor expansion of fractional order of time, a new model of fractional heat equation was established by Ezzatt and Karamany [16–18] and Sherief et al. [19]. Sherief and Abd El-Latief [20] studied the effect of the fractional order parameter and the variable thermal conductivity on a thermoelastic half-space. Due to thermal source, the effect of fractional order parameter on plane deformation in a thermoelastic medium was studied by Kumar et al. [21]. Recently, Abbas [22–25] studied the effect of fractional order on thermoelastic problems by using the eigenvalue approach.

The present work is an attempt to get a new picture of photothermoelastic theory with one relaxation time using the fractional calculus theories. Based on the eigenvalue approach and Laplace transformation, the governing nonhomogeneous equations are processed using an analytical-numerical technique. From the numerical results, the physical interpretations are given for the distribution of physical quantities observed in this study.

2. FORMULATION OF THE PROBLEM

A homogeneous semiconducting material is considered. The theoretical analysis of the transport processes in a semiconductor material involved in the study of coupled thermal, plasma and elastic waves simultaneously. The main variables are the distribution of temperature $T(\mathbf{r}, t)$, the density of carriers $n(\mathbf{r}, t)$ and the elastic displacement components $u_i(\mathbf{r}, t)$. When an ultrafast laser $Q(\mathbf{r}, t)$ falls on an isotropic, elastic and homogeneous semiconductor, the governing equations of motion, plasma and heat conduction under fractional order theory can be described as [2, 26–29]:

$$\mu(u_{i,jj} + u_{j,ij}) + \lambda u_{j,ij} - \gamma_n N_{,i} - \gamma_t \Theta_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (1)$$

$$D_{\mathrm{e}} N_{,jj} = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \delta \frac{\Theta}{\tau} - Q(\mathbf{r}, t), \quad (2)$$

$$K\Theta_{,jj} = -\frac{E_{\mathrm{g}}}{\tau} N + \left(1 + \frac{\tau_0^{\nu}}{\Gamma(\gamma+1)} \frac{\partial^{\nu}}{\partial t^{\nu}}\right) \times \left(\rho c_{\mathrm{e}} \frac{\partial \Theta}{\partial t} + \gamma_{\mathrm{t}} T_0 \frac{\partial u_{j,j}}{\partial t} - \delta_E Q(\mathbf{r}, t)\right), 0 < \nu \le 1, \quad (3)$$

the stress–strain relations are expressed as

$$\sigma_{ij} = \mu(u_{i,j} + u_{j,i}) + (\lambda u_{k,k} - \gamma_n N - \gamma_{\mathrm{t}} \Theta)\delta_{ij}, \quad (4)$$

where ρ is the medium density, τ_0 is the thermal relaxation time (for semiconductor $t_0 = 10^{-12} - 10^{-10}$ s), $N = n - n_0$, n_0 is the equilibrium carrier concentration, $\Theta = T - T_0$, T_0 is the reference temperature, u_i are the displacement components, λ, μ are the Lame's constants, $\gamma_n = (3\lambda + 2\mu) d_n$, d_n is the electronic deformation coefficient, $\gamma_{\mathrm{t}} = (3\lambda + 2\mu)\alpha_{\mathrm{t}}$, α_{t} is the coefficient of linear thermal expansion, σ_{ij} are the stress components, K is the thermal conductivity, D_{e} is the carrier diffusion coefficient, $\delta_E = E - E_{\mathrm{g}}$, E is the excitation energy, τ is the photogenerated carrier lifetime, and $\delta = \partial n_0 / \partial \Theta$ is the coupling parameter of thermal activation [30], c_{e} is the specific heat at constant strain, E_{g} is the semiconductor energy gap, $Q(\mathbf{r}, t) = \alpha \Phi(\mathbf{r}) f(t)$, α is the optical absorption coefficient, $\Phi(\mathbf{r})$ is the incident laser influence, $f(t)$ is the function of temporal modulation of the intensity of laser beam, t is the time, and $\mathbf{r}$ is the position vector. The different parameter values with a wide range $0 < \nu \le 1$ cover both conductivity, $\nu = 1$ for normal conductivity and $0 < \nu < 1$ for low conductivity. Let's consider an isotropic, homogeneous, semiconductor medium, occupying the region $x \ge 0$ where all the state functions depend only on the time t and the variable x. The x axis is taken perpendicular to the bounding plane of